Application of the FDTD Technique to Periodic Problems in Scattering and Radiation

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Abstract—**The finite-difference time-domain (FDTD) technique is applied to two-dimensional periodic problems in scattering and radiation. This method allows the direct and complete treatment of periodic scatterers and array having transverse or longitudinal inhomogeneities, and several examples are presented to illustrate the versatility of the technique. The FDTD grid needs only to be applied to a unit cell of the periodic structure, making the analysis very efficient from a computational point of view. The method is presently limited to broadside incidence, however.**

I. Introduction

The FDTD method is a useful computational technique for a wide variety of problems in electromagnetic radiation and scattering, especially when inhomogeneous materials are involved [1]–[3]. It is computationally intensive, and this requirement increases sharply with the electrical size of the problem, but this disadvantage is at least partially offset by increasingly powerful computers. Many problems in scattering and radiation, however, are periodic in one or two dimensions, and this periodicity can be used to simplify the analysis by considering an infinite-periodic array. This has been done via the moment method and other full-wave solutions for many problems [4], [5], but these techniques are generally limited to frequency domain analyses, and usually to geometries without transverse material inhomogeneities.

In this letter, we report on the extension of the FDTD technique to infinite-periodic arrays or scatterers. The usual rationale for the infinite-array approximation is that it offers a good model for the central elements of a large, but finite, array or scatterer. The FDTD grid needs only to be applied to a single-unit cell of the structure, making the analysis of a large structure much simpler and more computationally efficient than the modeling of the entire structure, especially since the unit cell of most periodic arrays and scatterers has linear dimensions less than one wavelength. But probably the main advantage of this method is that it can be used to model the effect of material inhomogeneities on the characteristics of the array or scatterer.

II. Theory

Consider a 2-D infinite-periodic scatterer illuminated by a plane wave at broadside incidence. Due to symmetry, the scattered field (solid arrows shown in Fig. 1) from this geometry will only propagate in the positive or negative y

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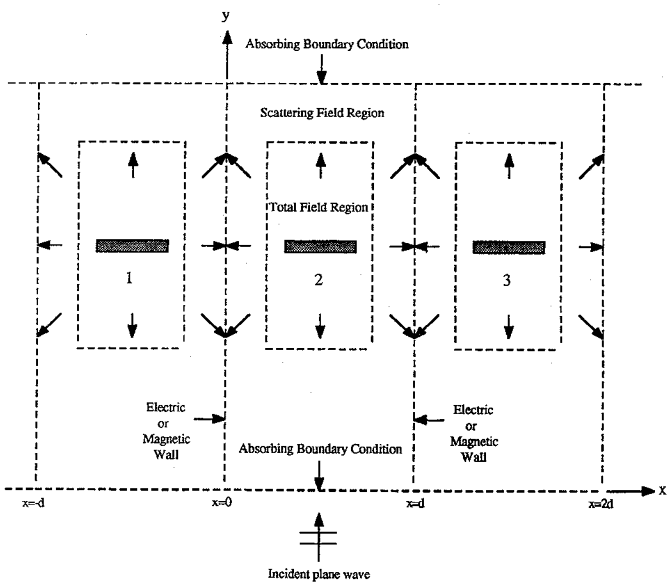

Fig. 1. Scattering from a 2-D infinite-periodic geometry illumidnated by a plane wave at broadside incidence.

directions, and either electric walls (TE case) or magnetic walls (TM case) will be formed at the unit cell boundaries at $x = 0$ and $x = d$. Thus, the infinite-array problem can be simplified to the single unit cell shown in Fig. 1. The upper and lower boundaries can be truncated using an absorbing boundary condition [6]. Then, the infinite array can be analyzed on the basis of a finite-size unit cell with the FDTD algorithm. For example, for TM incidence, the E_z field at the i, jth grid point can be expressed as

$$E_z^{n+1}(i,j) = E_z^n(i,j) + \frac{\Delta t}{\varepsilon\Delta}\left[H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) - H_y^{n+\frac{1}{2}}\left(i-\frac{1}{2},j\right) + H_x^{n+\frac{1}{2}}\left(i,j-\frac{1}{2}\right) - H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right)\right], \quad (1)$$

where Δ is the lattice increment, and Δt is the time increment. For the broadside TM case with propagation in the y direction, the magnetic field will satisfy

$$H_y^{n+\frac{1}{2}}\left(i-\frac{1}{2},j\right) = H_y^{n+\frac{1}{2}}\left(M-\frac{1}{2},j\right) \text{and}$$
$$H_y^{n+\frac{1}{2}}\left(1+\frac{1}{2},j\right) = H_y^{n+\frac{1}{2}}\left(M+\frac{1}{2},j\right) \quad (2)$$

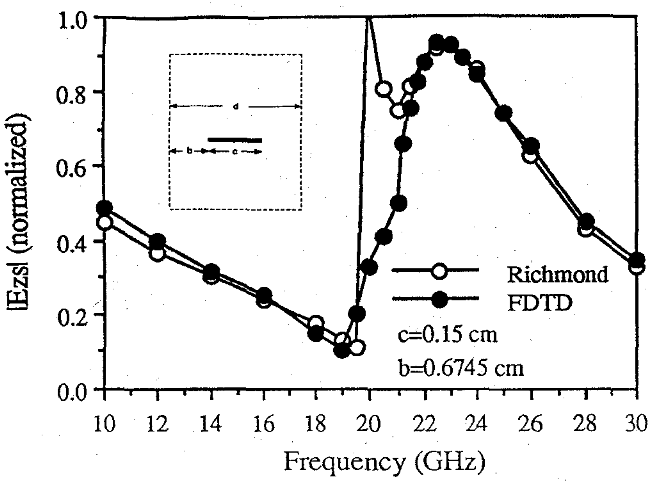

Fig. 2. Magnitude of the scattered E_z field from an infinite-strip grating.

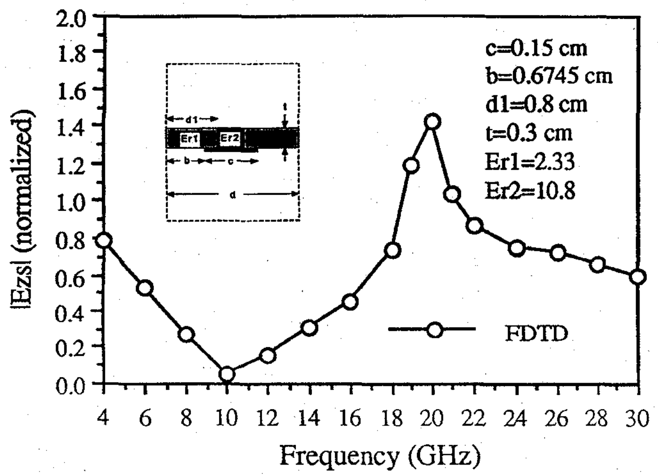

Fig. 3. Magnitude of the scattered E_z field from an infinite-strip with an inhomogeneous dielectric cover layer.

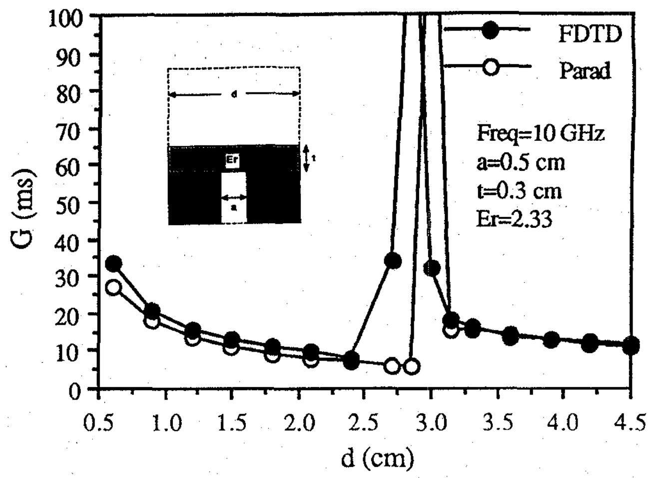

Fig. 4. Input conductance of an infinite-periodic slot array antenna.

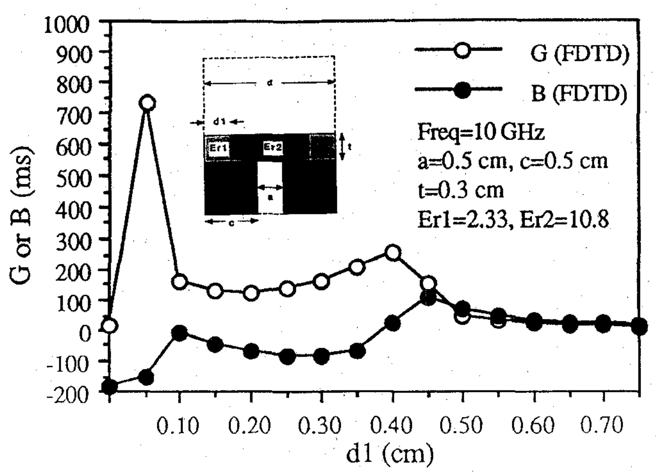

Fig. 5. Input admittance of an infinite-periodic slot array antenna with an inhomogeneous dielectric cover layer.

where M is the largest lattice grid number in the x direction. Equation (2) can be referred to as a periodic boundary condition. The remaining boundaries can be treated using the absorbing boundary condition. When a TE plane wave is incident, the magnetic walls are replaced with electric walls at $x = 0, d$, and a dual form of the previous equations can be obtained.

For a slot array antenna, a parrallel plate transmission line feed is used. The parallel plate separation, a, is chosen to allow a single TEM mode to propagate. An electric wall is used to close the slot in order to simulate a numerial phase reference for the input admittance. We also use a magnetic wall and a matched load to validate the FDTD solution. By so doing, the radiation problem is reduced to a problem involving only a transmission line terminated with an unkown load. Thus, the input admittance can be obtained by using the direct relationships between field and circuit quantities.

III. Results

We begin with the 2-D problem of TM scattering from an infinite-periodic strip grating. Fig. 2 shows the magnitude of the scattered E_z field, compared with an analytical solution from Richmond [5]. Good agreement is observed, and similar agreement has been obtained for the phase. It is interesting to notice that the sudden jump at the Wood's anomaly frequency at 20 GHz does not occur in the FDTD solution. But checking the magnitude and phase of the E_z field in the unit cell shows that the second lowest floquet mode starts propagating at 20 GHz, which is the definition of Wood's anomaly. This solution used 180×90 grid points, and about 6 minutes of CPU time per frequency point on a VAX 11/785. It is possible to obtain this data more efficiently for the entire frequency range by using a pulsed excitation followed by Fourier transformation.

Fig. 3 shows the effect of using an inhomogeneous dielectric cover on the strip grating. Note that near 10 GHz the scattered field amplitude drops to zero, and that a Wood's anomaly occurs near 20 GHz.

Results for the input admittance of an infinite-periodic slot array antenna are shown in Fig. 4, and compared with an analytical solution from Parad [7]. The real part of the input admittance is shown to be in good agreement, and similar agreement had been obtained for the imaginary part. Fig. 5 shows the effect of an inhomogeneous dielectric cover layer on the input admittance.

IV. Conclusion

The FDTD method has been applied to periodic scattering and radiation problems. The technique has the advantages of computational efficiency and the ability to handle arbitrary geometries and material inhomogeneities, but at the present time the method is limited to broadside incidence. For oblique incidence, the authors tried to implement the concept that the fields at one side of the unit cell are identical with the fields of the other side except for a time-delay. Apparently because the near-field higher order floquet modes propagate at different angles than that of the incident waves, this approach proved to be an unstable process. It is also possible to treat periodic problems as modeled in waveguide simulators, but such cases correspond to actual infinite arrays or scatterers only for specific combinations of scan angle and frequency.

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